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SHORTER COMMUNICATIONS

STUDY OF THE ANALOGY BETWEEN HEAT AND MASS TRANSFER TO THE WALL OF A STIRRED TANK

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NOMENCLATURE

A_c, A_m, local coefficients [equations (4) and (6)] [cm^{-1}];
C_p, specific heat of the liquid [cal/g°C];
D, turbine diameter [cm];
h, level of the point of measurement from the bottom of the tank [cm];
h_c, heat-transfer coefficient [$cal/cm^2.s.°C$];
H_a, height of the median level of the agitator from the bottom of the tank [cm];
k_L, mass-transfer coefficient;
m, exponent [equation (6)];
n, exponent [equation (4)];
N, rate of rotation of the agitator [s^{-1}];
T, tank diameter [cm];
$\bar{U}$, mean fluid velocity near the wall, outside the boundary layer [cm/s].

Greek symbols

Δ, mass diffusivity [cm^2/s];
ρ, liquid density [g/cm^3];
μ, liquid viscosity [P];
λ, thermal conductivity [$cal/cm^2.s.°C/cm$];
σ, factor of proportionality between $\bar{U}$ and πND.

Dimensionless groups

j_m, j_c, Colburn factors for mass and heat transfer [equations (1) and (2)];
Pr, Prandtl number $Pr = C_p\mu/\lambda$;
Re_a, agitator Reynolds number $= Re_a = ND^2\rho/\mu$;
Sc, Schmidt number, $Sc = \mu/\rho.\Delta$;
Sh_T, Sherwood number $Sh_T = k_L T/\Delta$;
Nu_T, Nusselt number $Nu_T = h_c T/\lambda$.

1. INTRODUCTION

IN THE case of mechanical stirred tanks, it is useful to confirm the validity of the Colburn analogy between mass and heat transfer: (i) from a practical point of view, if the above analogy held it would then be possible to interchange experimental results from one type of transfer to the other, for example to predict heat-transfer rates by means of mass-transfer results easily obtained by the electrochemical method used in earlier work [1];

(ii) from a fundamental point of view the concept of analogy would yield useful information on cases of complex and turbulent velocity field.

Three papers concerning the various types of transfer between a liquid contained in a mechanical stirred tank and the wall of the vessel have been published [2–4]: all concluded to a complete analogy between momentum, mass and heat transfer. However, the following comments should be noted here: (i) Askew and Beckmann [2–3] have only studied baffled stirred tanks. Their experimental local results were independent of the dimension of the active portion of the surface element: this may be questionable. They further compared these results with "theoretical" prediction derived for a flat surface. The agreement was not quite good probably due to entrance effects related to the development of the boundary layer.

(ii) Mizushina *et al.* [4] have investigated the momentum and mass transfer analogy only in an unbaffled stirred tank. Comparing their results with the work of Chilton *et al.* [5] for heat transfer in a similar equipment, these authors concluded that the analogy between the three types of transfer was verified.

In preliminary studies the authors of the following paper have investigated flow patterns inside baffled and unbaffled stirred tanks [6–7]. An electrochemical method giving the mass-transfer coefficient between a fixed sphere and the surrounding fluid was used and an estimated value of the mean fluid velocity $\bar{U}$ near the wall of the tank but outside the boundary layer was proposed [6–7]. The authors have further investigated separately local heat and mass transfer to the wall of stirred tanks [8–9]. Various values of D/T, various distance $(H_a - h)$ changing H_a and various rate of rotation of the standard turbine have been used. Overall heat and mass transfer were then obtained by integration of the local values. Results were presented in terms of the usual dimensionless groups Re_a, Pr, Nu_T, Sc and Sh_T [8–9].

Using the above information it is proposed in this work to analyse the transfer mechanisms in the light of the measured hydrodynamic to calculate j_m and j_c, the Colburn factors [10], and to discuss the analogies between the various transfers. This work will concern local and overall coefficients.

2. EXPERIMENTAL RESULTS

2.1 *Local values*

Using the heat- and mass-transfer coefficient measured for each distance $(H_a - h)$, and the mean fluid velocity $\bar{U}$ the Colburn coefficients may be written [10]

$$j_m = \frac{k_L}{\bar{U}} Sc^{2/3} \quad (1)$$

$$j_c = \frac{h_c}{\rho C_p \bar{U}} Pr^{2/3} \quad (2)$$

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Table 1. j_m and j_c values for unbaffled stirred tanks

T (cm)	D (cm)	$\frac{D}{T}$	$\frac{H_a-h}{T}$	σ	A_m or A_c (cm^{-1})	$\frac{A_mD}{\sigma\pi}$ or $\frac{A_cD}{\sigma\pi}$	$(n-1)$ or $(m-1)$	j_m or j_c
28·2	6·2	0·220	0·000	0·111	0·0351	0·625	−0·41	$j_m = 0{\cdot}625 Re_a^{-0{\cdot}41}$
			0·145	0·111	0·0321	0·571	−0·41	$j_m = 0{\cdot}571 Re_a^{-0{\cdot}41}$
	9·4	0·333	0·000	0·195	0·0323	0·495	−0·39	$j_m = 0{\cdot}495 Re_a^{-0{\cdot}39}$
			0·039	0·202	0·0321	0·475	−0·39	$j_m = 0{\cdot}475 Re_a^{-0{\cdot}39}$
			0·145	0·188	0·0297	0·473	−0·39	$j_m = 0{\cdot}473 Re_a^{-0{\cdot}39}$
			0·287	0·181	0·0256	0·423	−0·39	$j_m = 0{\cdot}423 Re_a^{-0{\cdot}39}$
			0·358	0·174	0·0258	0·444	−0·39	$j_m = 0{\cdot}444 Re_a^{-0{\cdot}39}$
	13·4	0·475	0·000	0·276	0·0224	0·346	−0·35	$j_m = 0{\cdot}346 Re_a^{-0{\cdot}35}$
			0·145	0·267	0·0202	0·323	−0·35	$j_m = 0{\cdot}323 Re_a^{-0{\cdot}35}$
28·8	9·6	0·333	0·000	0·195	0·0325	0·509	−0·37	$j_c = 0{\cdot}509 Re_a^{-0{\cdot}37}$
			0·038	0·202	0·0328	0·496	−0·37	$j_c = 0{\cdot}496 Re_a^{-0{\cdot}37}$
			$0{\cdot}142_5$	0·188	0·0289	0·470	−0·37	$j_c = 0{\cdot}470 Re_a^{-0{\cdot}37}$
			0·351	0·174	0·0269	0·473	−0·37	$j_c = 0{\cdot}473 Re_a^{-0{\cdot}37}$
			0·477	0·160	0·0250	0·477	−0·37	$j_c = 0{\cdot}477 Re_a^{-0{\cdot}37}$

Table 2. j_m and j_c values for baffled stirred tanks

T (cm)	D (cm)	$\frac{D}{T}$	$\frac{H_a-h}{T}$	σ	A_m or A_c (cm^{-1})	$\frac{A_mD}{\sigma\pi}$ or $\frac{A_cD}{\sigma\pi}$	$(n-1)$ or $(m-1)$	j_m or j_c
28·2	6·2	0·220	0·000	$0{\cdot}100_5$	0·0375	0·736	−0·38	$j_m = 0{\cdot}736 Re_a^{-0{\cdot}38}$
			0·145	0·085	0·0317	0·736	−0·38	$j_m = 0{\cdot}736 Re_a^{-0{\cdot}38}$
	9·4	0·333	0·000	0·218	0·0345	0·472	−0·35	$j_m = 0{\cdot}472 Re_a^{-0{\cdot}35}$
			0·039	0·210	0·0321	0·456	−0·35	$j_m = 0{\cdot}456 Re_a^{-0{\cdot}35}$
			0·145	0·188	0·0273	0·435	−0·35	$j_m = 0{\cdot}435 Re_a^{-0{\cdot}35}$
			0·287	0·160	0·0228	0·426	−0·35	$j_m = 0{\cdot}426 Re_a^{-0{\cdot}35}$
			0·351	$0{\cdot}122_5$	0·0204	0·500	−0·35	$j_m = 0{\cdot}500 Re_a^{-0{\cdot}35}$
	13·4	0·475	0·000	0·321	0·0395	0·525	−0·35	$j_m = 0{\cdot}525 Re_a^{-0{\cdot}35}$
			0·145	0·267	0·0303	0·482	−0·35	$j_m = 0{\cdot}482 Re_a^{-0{\cdot}35}$
28·8	9·6	0·333	0·000	0·218	0·0404	0·565	−0·35	$j_c = 0{\cdot}565 Re_a^{-0{\cdot}35}$
			$0{\cdot}142_5$	0·189	0·0295	0·476	−0·35	$j_c = 0{\cdot}476 Re_a^{-0{\cdot}35}$
			0·347	0·123	0·0206	0·511	−0·35	$j_c = 0{\cdot}511 Re_a^{-0{\cdot}35}$

Table 3. Overall transfer coefficients ($D/T = 0{\cdot}333$)

	Mass Transfer	Heat Transfer
Unbaffled tank	$Sh_T.Sc^{-1/3} = \left[0{\cdot}91_4 - 0{\cdot}28_8\left\{\left(\frac{H_a}{T} - 1{\cdot}02\right)^2 + \left(\frac{H_a}{T} - 0{\cdot}02\right)\right\}\right]Re_a^{0{\cdot}61}$	$Nu_T.Pr^{-1/3} = \left[0{\cdot}95_1 - 0{\cdot}24_5\left\{\left(\frac{H_a}{T} - 1{\cdot}02\right)^2 + \left(\frac{H_a}{T} - 0{\cdot}02\right)^2\right\}\right]Re_a^{0{\cdot}63}$
Baffled tank	$Sh_T.Sc^{-1/3} = \left[0{\cdot}41_5 - 1{\cdot}14\frac{H_a}{T}\left(\frac{H_a}{T} - 1\right)\right]Re_a^{0{\cdot}65}$	$Nu_T.Pr^{-1/3} = \left[1{\cdot}14 - 0{\cdot}77\left\{\left(\frac{H_a}{T} - 1\right)^2 + \left(\frac{H_a}{T}\right)^2\right\}\right]Re_a^{0{\cdot}65}$

Under turbulent conditions, the mean velocity $\bar{U}$ of the liquid outside the boundary layer is proportional to πND, [8, 9]. Furthermore, the relation between the mass-transfer coefficient k_L and other physical and geometrical parameters may be written [6]

$$k_L/\Delta . Sc^{1/3} = A_m Re_a^n. \tag{3}$$

While the heat-transfer coefficient h_c may be expressed by [7]

$$h_c/\lambda . Pr^{1/3} = A_c Re_a^n. \tag{4}$$

Relations (1) and (2) then become

$$j_m = \frac{A_m D}{\sigma\pi} Re_a^{n-1} \tag{5}$$

$$j_c = \frac{A_c D}{\sigma\pi} Re_a^{m-1}. \tag{6}$$

Here, changes in the fluid viscosity between the wall and the core of the liquid have been neglected.

This sort of presentation allows to consider local transfer phenomena in terms of $\bar{U}$ instead of using the dimensions of the turbine agitating the fluid. Moreover as the difference between the size of the tanks is very small (< 2 per cent) mass- and heat-transfer results can be compared.

2.1.1 *Unbaffled tank.* Calculated values of j_m and j_c using equation (5) and (6) are given in Table 1. These results correspond to local transfer values corrected for entrance effects. Figure 1 shows the Colburn factors plotted against Re_a. It can be noticed that: (i) j_c is always greater than j_m, maximum difference being in the range of 41–49 per cent; (ii) for a given value of Re_a, j_c remains practically constant (within 6 per cent) for all values of the other parameters, and (iii) no significant dependance of j_m on D/T can be noted, the maximum difference being 26 per cent; when $D/T = 0{\cdot}333$, j_m seems to decrease when $(H_a - h)$ increases.

2.1.2 *Baffled tank.* Colburn factors for baffled tanks are given in Table 2 and plotted against Re_a in Fig. 2. In this case, it can be seen that: (i) j_m and j_c values are not too widely dispersed; when Re_a is in the range of 5×10^3 to 5×10^4 the maximum difference between all values is less than 30 per cent; (ii) j_m values corresponding to $D/T = 0{\cdot}220$ seem larger than for the two other turbines; (iii) no systematic dependance of j on $(H_a - h)$ can be noticed; (iv) the maximum deviation between j_c and j_m is about 20 per cent under the same experimental conditions.

Let us recall here that the use of $\bar{U}$, the mean fluid velocity, gives only an approximated value of the transfer coefficient at the wall. In conclusion however it should be noted that under the same geometrical conditions ($D/T = 0{\cdot}333$): (a) in unbaffled tanks j_c is 30–35 per cent larger than j_m, and (b) in baffled tanks j_c is at the most 20 per cent larger than j_m.

2.2 *Overall values*

Overall heat and mass transfer were obtained by integration of the local values [8, 9]; the main correlations are shown in Table 3. Comparison of the results shows that: (i) in unbaffled tanks, the ratio of heat to mass transfer is equal to $1{\cdot}09 Re_a^{0{\cdot}02}$ and hence increases with Re_a; for Re_a of order 3×10^3 to 5×10^4, the above ratio is in the range of 1·28–1·32, and (ii) in baffled tanks, depending on the value of H_a/T, heat transfer is from 3·5–8·5 per cent larger than mass transfer.

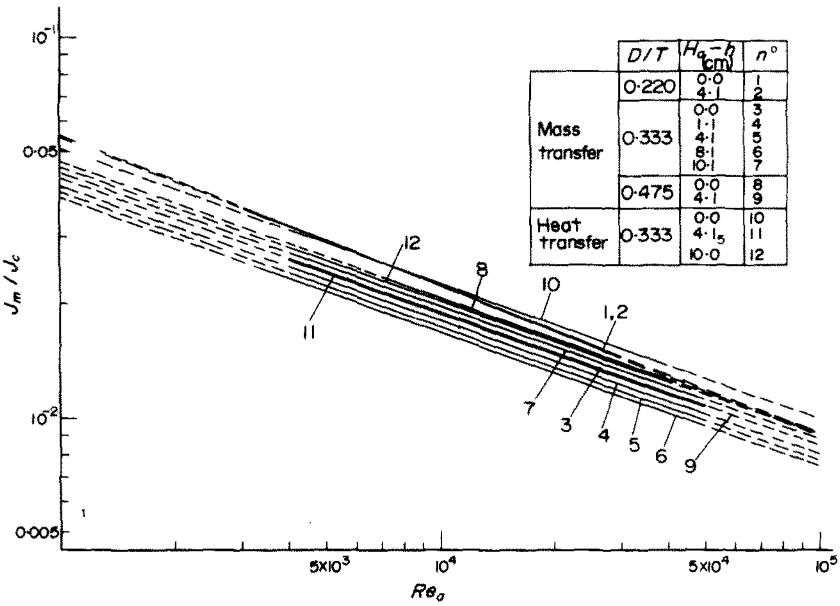

FIG. 1. Plot of j_m and j_c vs Re_a for unbaffled stirred tanks.

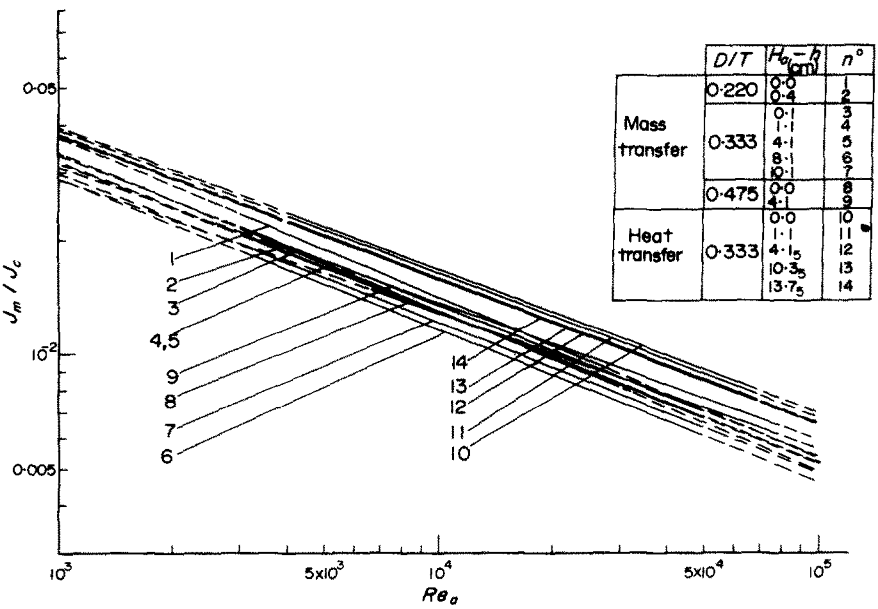

FIG. 2. Plot of j_m and j_c vs Re_a for baffled stirred tanks.

3. DISCUSSION

The above results indicate that for all practical purposes, the analogy between heat and mass transfer is verified in baffled tanks as opposed to unbaffled tanks where heat transfer is more pronounced. An explanation for this behaviour can be found by noting that, for the experimental conditions of this work, the Prandtl number was around 7 or 8 while the Schmidt number was in the range of 1500–2000. The thickness of the boundary layer was therefore quite different for the two types of transfer: the main resistance to heat transfer was located in a layer about six times thicker than for mass transfer. Heat transfer was therefore more sensitive to the various perturbation caused either by erratic turbulence or by the regular motion of the blades of the turbine particularly if those perturbations were not important enough to disturb the whole hydrodynamic boundary layer. In the case of baffled tanks the above phenomenon was damped to a certain extent due to the controlling importance of turbulence which influences similarly both types of transfer but nevertheless heat transfer remains somewhat larger.

It has already been reported [6] that unbaffled stirred tanks equipped with a large agitator seem to behave like baffled tanks and that, in a reciprocal manner baffled tanks equipped with a small agitator (D/T small) resemble unbaffled tanks, i.e. there is no sharp discontinuity between both systems. The results presented here corroborate the above idea: for example as can be seen on graph 1, j_m for $D/T = 0{\cdot}475$ is larger than for the two other turbines and yet not very different from j_m calculated for baffled tanks.

In order to explain the general differences between the two systems considered above the pumping capacity and its influence on the flow pattern has to be taken into account. For baffled tanks, the pumping capacity is important and the flow leaves the turbine blades like a jet which impinges on the vertical wall of the tank. For unbaffled tanks, the pumping capacity is less important and the fluid velocity is tangential. The difference between these two mixing mechanisms becomes however more or less important according to the size of the turbine. In the case of baffled tanks equipped with a small turbine the liquid jet does not reach the wall due to the drag and the results presented here indicated that the exponent of Reynolds number decreases.

In the case of unbaffled tanks equipped with a large agitator, the importance of the pumping capacity increases particularly when the blade gets closer to the wall; the exponent of Reynolds number is the same as in the case of the baffled tank ($D/T = 0{\cdot}475$). It can be then understood why Mizushina *et al.* [4] have verified the analogy between heat and mass transfer in an unbaffled tank: the tank was indeed equipped with a large agitator ($D/T = 0{\cdot}66$).

4. CONCLUSIONS

It was shown that for baffled tanks the Colburn factors for heat and mass transfer to the wall can be approximated by the same equation. For unbaffled tanks, heat transfer was significantly larger due to the pronounced difference between the values of the Prandtl and the Schmidt numbers and hence to the larger influence of perturbations on the thicker heat-transfer boundary layer. It should be noticed that the exponents of the Prandtl and Schmidt numbers were set equal to 1/3; in fact more sophisticated studies would be required to point out the influence of erratic phenomenon on that exponent [12–13].

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REFERENCES

1. A. Le Lan, Thèse Docteur ès-Sciences, Toulouse (1973).
2. W. S. Askew and R. B. Beckmann, *I/EC Process. Des. Dev.* **4**, 311 (1965).
3. W. S. Askew and R. B. Beckmann, *I/EC Process. Des. Dev.* **5**, 268 (1966).
4. T. Mizushina, R. Ito, S. Hiraoka, A. Ibusuki and I. Sagagushi, *J. Chem. Engng Japan* **2**, 89 (1969).
5. T. H. Chilton, T. B. Drew and R. H. Jebens, *Ind. Engng Chem.* **36**, 510 (1944).
6. A. Le Lan, H. Gibert and H. Angelino, *Chem. Engng Sci.* **27**, 1979 (1972).
7. A. Le Lan and H. Angelino, *Chem. Engng Sci.* **29**, 907 (1974).
8. A. Le Lan and H. Angelino, *Chem. Engng Sci.* **29**, 1557 (1974).
9. A. Le Lan, C. Laguerie and H. Angelino, *Chem. Engng Sci.* To be published.
10. A. P. Colburn, *Trans. Am. Instn Chem. Engrs* **29**, 174 (1933).
11. R. Gardon and J. C. Akfirat, *Int. J. Heat Mass Transfer* **8**, 1261 (1965).
12. V. G. Levich, *Physicochemical Hydrodynamics.* Prentice Hall, Englewood Cliffs (1962).
13. J. T. Davies, *Turbulence Phenomena.* Academic Press, New York (1972).

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LOCAL HEAT-TRANSFER COEFFICIENTS ON THE ROTATING DISK IN STILL AIR

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NOMENCLATURE

A, area of h-calorimeter front surface [m^2];
B, relative error of h-calorimeter cooling rate due to errors in temperature measurements [%/deg];
c, specific heat [J/kg deg];
C, constant of h-calorimeter, Gc/A [J/m^2deg];
g, acceleration of gravity [m/s^2];
G, mass of h-calorimeter [kg];
Gr, Grashof number, $\beta g(T_w - T_\infty)L^3/\nu^2$;
h, local heat-transfer coefficient [W/m^2deg];
k, thermal conductivity [W/m deg];
L, average equivalent height of h-calorimeter surface as measured from bottom edge of disk [m];